

**The equation for determining the caustic point  
associated with the *inflection* points in the {001} planes**

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$$f = c_0 + c_1 \cos 4\varphi + c_2 \cos^2 4\varphi + c_3 \cos^3 4\varphi = 0$$

where

$$\begin{aligned} c_0 = & -(64a^{18} - 384a^{17} - 432b^2a^{16} - 864ba^{16} - 624a^{16} + 2400b^2a^{15} + 4800ba^{15} + \\ & 8032a^{15} + 1287b^4a^{14} + 5148b^3a^{14} + 4650b^2a^{14} - 996ba^{14} - 11001a^{14} - \\ & 6570b^4a^{13} - 26280b^3a^{13} - 67164b^2a^{13} - 81768ba^{13} - 54282a^{13} - 2235b^6a^{12} - \\ & 13410b^5a^{12} - 16584b^4a^{12} + 23064b^3a^{12} + 168153b^2a^{12} + 254418ba^{12} + 187954a^{12} + \\ & 10308b^6a^{11} + 61848b^5a^{11} + 203352b^4a^{11} + 401088b^3a^{11} + 319236b^2a^{11} + 1224ba^{11} - \\ & 107136a^{11} + 2511b^8a^{10} + 20088b^7a^{10} + 35262b^6a^{10} - 69660b^5a^{10} - 584337b^4a^{10} - \\ & 1497972b^3a^{10} - 1723644b^2a^{10} - 851376ba^{10} - 258120a^{10} - 10110b^8a^9 - 80880b^7a^9 - \\ & 318484b^6a^9 - 778584b^5a^9 - 804702b^4a^9 + 461992b^3a^9 + 1500960b^2a^9 + 864432ba^9 + \\ & 336608a^9 - 1899b^{10}a^8 - 18990b^9a^8 - 46476b^8a^8 + 83952b^7a^8 + 925113b^6a^8 + \\ & 3099222b^5a^8 + 5148558b^4a^8 + 3930576b^3a^8 + 897240b^2a^8 - 156000ba^8 - 113184a^8 + \\ & 6264b^{10}a^7 + 62640b^9a^7 + 288240b^8a^7 + 802560b^7a^7 + 1050648b^6a^7 - 722544b^5a^7 - \\ & 4065408b^4a^7 - 4481472b^3a^7 - 1570176b^2a^7 + 145920ba^7 + 59136a^7 + 957b^{12}a^6 + \\ & 11484b^{11}a^6 + 37158b^{10}a^6 - 49500b^9a^6 - 802467b^8a^6 - 3210552b^7a^6 - 7003208b^6a^6 - \\ & 8482656b^5a^6 - 4954512b^4a^6 - 691840b^3a^6 + 173184b^2a^6 - 245760ba^6 - 46720a^6 - \\ & 2310b^{12}a^5 - 27720b^{11}a^5 - 148548b^{10}a^5 - 469080b^9a^5 - 812454b^8a^5 - 120432b^7a^5 + \\ & 2582976b^6a^5 + 5176608b^5a^5 + 4315968b^4a^5 + 1517568b^3a^5 + 195840b^2a^5 + 74496ba^5 - \\ & 297b^{14}a^4 - 4158b^{13}a^4 - 16656b^{12}a^4 + 16344b^{11}a^4 + 394227b^{10}a^4 + 1757406b^9a^4 + \\ & 4452462b^8a^4 + 7191072b^7a^4 + 7433328b^6a^4 + 4851648b^5a^4 + 2147136b^4a^4 + 794112b^3a^4 + \\ & 187008b^2a^4 + 420b^{14}a^3 + 5880b^{13}a^3 + 36312b^{12}a^3 + 129984b^{11}a^3 + 298404b^{10}a^3 + \\ & 460200b^9a^3 + 499200b^8a^3 + 450624b^7a^3 + 462464b^6a^3 + 459264b^5a^3 + 260352b^4a^3 + \\ & 45568b^3a^3 + 45b^{16}a^2 + 720b^{15}a^2 + 3378b^{14}a^2 - 3108b^{13}a^2 - 91731b^{12}a^2 - \\ & 414996b^{11}a^2 - 1005180b^{10}a^2 - 1511664b^9a^2 - 1490472b^8a^2 - 1019520b^7a^2 - 533376b^6a^2 - \\ & 218112b^5a^2 - 49536b^4a^2 - 18b^{16}a - 288b^{15}a - 1740b^{14}a - 4200b^{13}a + 3150b^{12}a + \\ & 46536b^{11}a + 130080b^{10}a + 189360b^9a + 157920b^8a + 76800b^7a + 25344b^6a + 6912b^5a - \\ & b^{18} - 18b^{17} - 108b^{16} - 96b^{15} + 2043b^{14} + 12474b^{13} + 37266b^{12} + 68112b^{11} + 80568b^{10} + \\ & 63136b^9 + 34272b^8 + 13824b^7 + 3456b^6) \end{aligned}$$

$$\begin{aligned}
c_1 = & 3(a-b-2)(a+b)(64a^{16} - 256a^{15} - 368b^2a^{14} - 736ba^{14} - 624a^{14} + \\
& 1344b^2a^{13} + 2688ba^{13} + 3648a^{13} + 901b^4a^{12} + 3604b^3a^{12} + 4366b^2a^{12} + 1524ba^{12} - \\
& 2955a^{12} - 2964b^4a^{11} - 11856b^3a^{11} - 20216b^2a^{11} - 16720ba^{11} - 6676a^{11} - \\
& 1226b^6a^{10} - 7356b^5a^{10} - 12140b^4a^{10} + 480b^3a^{10} + 25974b^2a^{10} + 31372ba^{10} + \\
& 20336a^{10} + 3548b^6a^9 + 21288b^5a^9 + 46112b^4a^9 + 42528b^3a^9 + 23564b^2a^9 + \\
& 18840ba^9 - 7720a^9 + 1015b^8a^8 + 8120b^7a^8 + 18242b^6a^8 - 4228b^5a^8 - 62505b^4a^8 - \\
& 70260b^3a^8 - 80552b^2a^8 - 118472ba^8 - 39152a^8 - 2472b^8a^7 - 19776b^7a^7 - \\
& 50192b^6a^7 - 24288b^5a^7 + 57880b^4a^7 + 24288b^3a^7 - 3232b^2a^7 + 91136ba^7 + 52224a^7 - \\
& 524b^{10}a^6 - 5240b^9a^6 - 16280b^8a^6 - 4480b^7a^6 + 43812b^6a^6 - 26536b^5a^6 - \\
& 276672b^4a^6 - 342688b^3a^6 - 164864b^2a^6 - 69760ba^6 - 25728a^6 + 984b^{10}a^5 + 9840b^9a^5 + \\
& 25568b^8a^5 - 31616b^7a^5 - 248776b^6a^5 - 388784b^5a^5 - 129840b^4a^5 + 222848b^3a^5 + \\
& 234496b^2a^5 + 79488ba^5 + 8064a^5 + 163b^{12}a^4 + 1956b^{11}a^4 + 8402b^{10}a^4 + 12300b^9a^4 + \\
& 4243b^8a^4 + 83000b^7a^4 + 433488b^6a^4 + 888464b^5a^4 + 917280b^4a^4 + 475008b^3a^4 + \\
& 96256b^2a^4 + 1920ba^4 - 196b^{12}a^3 - 2352b^{11}a^3 - 5816b^{10}a^3 + 28080b^9a^3 + \\
& 204668b^8a^3 + 549472b^7a^3 + 809184b^6a^3 + 705280b^5a^3 + 366592b^4a^3 + 115456b^3a^3 + \\
& 23808b^2a^3 - 26b^{14}a^2 - 364b^{13}a^2 - 2060b^{12}a^2 - 5792b^{11}a^2 - 8186b^{10}a^2 - \\
& 8292b^9a^2 - 25328b^8a^2 - 71200b^7a^2 - 95872b^6a^2 - 55680b^5a^2 - 4736b^4a^2 + \\
& 5376b^3a^2 + 12b^{14}a + 168b^{13}a + 896b^{12}a + 2016b^{11}a + 60b^{10}a - 9256b^9a - 19176b^8a - \\
& 15744b^7a - 4096b^6a - 384b^5a - 1152b^4a + b^{16} + 16b^{15} + 94b^{14} + 196b^{13} - 383b^{12} - \\
& 3140b^{11} - 7784b^{10} - 9992b^9 - 7088b^8 - 3456b^7 - 2304b^6 - 1152b^5)
\end{aligned}$$

$$\begin{aligned}
c_2 = & -3(a-b-2)^2(a+b)^2(64a^{14} - 128a^{13} - 304b^2a^{12} - 608ba^{12} - 368a^{12} + \\
& 544b^2a^{11} + 1088ba^{11} + 800a^{11} + 579b^4a^{10} + 2316b^3a^{10} + 3378b^2a^{10} + 2124ba^{10} - 733a^{10} - \\
& 902b^4a^9 - 3608b^3a^9 + 508b^2a^9 + 8232ba^9 + 7194a^9 - 557b^6a^8 - 3342b^5a^8 - \\
& 7564b^4a^8 - 7976b^3a^8 - 1257b^2a^8 + 4526ba^8 - 294a^8 + 728b^6a^7 + 4368b^5a^7 + \\
& 640b^4a^7 - 26560b^3a^7 - 63368b^2a^7 - 61968ba^7 - 18576a^7 + 278b^8a^6 + 2224b^7a^6 + \\
& 7052b^6a^6 + 11176b^5a^6 + 8390b^4a^6 + 568b^3a^6 + 552b^2a^6 + 6048ba^6 + 864a^6 - \\
& 292b^8a^5 - 2336b^7a^5 - 6344b^6a^5 - 5360b^5a^5 + 28892b^4a^5 + 107696b^3a^5 + \\
& 137808b^2a^5 + 70848ba^5 + 12960a^5 - 66b^{10}a^4 - 660b^9a^4 - 3168b^8a^4 - 9504b^7a^4 - \\
& 17594b^6a^4 - 16860b^5a^4 - 660b^4a^4 + 16832b^3a^4 + 20448b^2a^4 + 12960ba^4 + 2592a^4 + \\
& 56b^{10}a^3 + 560b^9a^3 + 4736b^8a^3 + 24448b^7a^3 + 75640b^6a^3 + 149200b^5a^3 + \\
& 184080b^4a^3 + 126336b^3a^3 + 36288b^2a^3 + 7b^{12}a^2 + 84b^{11}a^2 + 754b^{10}a^2 + \\
& 4460b^9a^2 + 16407b^8a^2 + 39000b^7a^2 + 61512b^6a^2 + 63648b^5a^2 + 40736b^4a^2 + \\
& 14016b^3a^2 + 1728b^2a^2 - 6b^{12}a - 72b^{11}a - 340b^{10}a - 760b^9a - 742b^8a - 368b^7a - \\
& 1232b^6a - 2624b^5a - 1632b^4a - b^{14} - 14b^{13} - 84b^{12} - 280b^{11} - 605b^{10} - 1122b^9 - 2230b^8 - \\
& 3584b^7 - 3168b^6 - 864b^5 + 288b^4)
\end{aligned}$$

$$\begin{aligned}
c_3 = & (a-b-2)^3(a+b)^3(8a^6 - 15b^2a^4 - 30ba^4 + 9a^4 + 6b^4a^2 + 24b^3a^2 + 114b^2a^2 + \\
& 180ba^2 + 108a^2 + b^6 + 6b^5 + 21b^4 + 44b^3 + 36b^2)^2
\end{aligned}$$


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