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Wave Motion 40 (2004) 173–190

WAVE MOTION

www.elsevier.com/locate/wavemoti

Space of degeneracy in the Stroh eigensystem and surface waves in transversely isotropic elastic media

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Received 22 April 2003 ; received in revised form 18 March 2004; accepted 23 March 2004

Abstract

Surface waves in anisotropic elastic media can be described by the linear combination of eigenvectors of the Stroh fundamental matrix **N**. The matrix **N** can admit two types of degeneracies: semisimple and nonsemisimple. The present study is to show that in the transversely isotropic media the degeneracies can span a two-dimensional space for either type of the degeneracies. Relationship between the spaces of degeneracy and the surface waves is discussed and analytical results for the surface wave solutions admitting generalized eigenvectors is presented with examples for the β-configuration in the transversely isotropic elastic media.

Keywords: Stroh formalism; Degeneracy; Surface waves; Anisotropic media

1. Introduction

Theoretical foundation for the existence of surface waves in anisotropic elastic media has been well established [1–5]. It was shown that there are large variety of surface waves which can be either subsonic or supersonic. An important development is about the existence of the so-called one-component supersonic surface waves where the degeneracy in the Stroh formalism plays a central role [6–9]. Recently, there are some studies on the classification of various types of surface waves with respect to the degeneracies in the Stroh formalism [10–12].

As for the subsonic surface waves, the influence of degeneracies has not yet been investigated thoroughly. The present work is to examine the existence of various degeneracies in the Stroh formalism in the so-called β-configuration, a simple surface configuration in the transversely isotropic media.

As a two-dimensional steady-state problem, we define a reference plane **R** spanned by $(\mathbf{m}, \mathbf{n})$, where **m** is the propagation direction and **n** the inward surface normal (Fig. 1) for a half-infinite elastic medium. The displacement vector **u** and the equation of motion can be written as [1,2]

$$\mathbf{u} = \mathbf{a}_\alpha \exp(ikz_\alpha), \quad z_\alpha = \mathbf{m}\cdot\mathbf{x} + p_\alpha \mathbf{n}\cdot\mathbf{x}, \tag{1.1}$$

$$\{Q + p_\alpha(R + R^{\mathrm{T}}) + p_\alpha^2 T\}\mathbf{a}_\alpha = \rho v^2 \mathbf{a}_\alpha, \tag{1.2}$$

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doi:10.1016/j.wavemoti.2004.03.001