

# Analytical determination of parabolic points on slowness surface and swallowtail points on wave surface of cubic crystals

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Slowness surface for bulk wave propagation in anisotropic media can be divided into concave, saddle and convex regions by the parabolic lines. When a parabolic line crosses a symmetry plane, it leaves either an inflection point or a parabolic point. Surface normal at these points is associated with cuspidal point and swallowtail point, respectively, on the wave surface and in phonon focusing patterns. By examining the degeneracies in the Stroh eigenvalue equation, we have calculated the cuspidal points in cubic crystals analytically. In this work, the parabolic point and its surface normal are discussed. The main idea is to establish a connection between the parabolic point and the extraordinary transonic state that is related to a degeneracy with a multiplicity of four in the Stroh eigenvalue equation. Such a connection yields a series of simple expressions, which determine the locations of parabolic points and the corresponding swallowtail points. The result is demonstrated using phonon focusing patterns of cubic crystals, and the method also provides a tool for general discussion of the slowness surface geometry.

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## 1. Introduction

Slowness surface, defined by the Christoffel equation for the bulk wave propagation in anisotropic elastic media, exhibits many interesting features (Musgrave 1970; Auld 1973). It is well known that each sheet of the slowness surface can be divided into concave, saddle and convex regions separated by the so-called parabolic line along which the Gaussian curvature is zero (Every 1981; Wolfe 1998). There are also various kinds of acoustic axes where two slowness sheets meet. A proper way to examine the geometry of slowness surfaces is to study the signatures left by the parabolic lines at symmetry planes. In a symmetry plane, the slowness surface may have points with either in-plane zero curvature or ex-plane zero curvature. The former refers to an inflection point marking a fold on the slowness surface and the latter refers to a parabolic point as shown in figure 1.

Wave surface, defined by the envelope of the slowness surface, describes group velocity for bulk waves. The geometry of the wave surface can be studied by examining either the Gauss map theoretically or phonon focusing patterns

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