Structure of phonon focusing patterns in tetragonal crystals

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A set of critical conditions for the characteristic caustic points in the phonon focusing patterns in tetragonal crystals is formulated. A caustic line segment in the focusing pattern is generally associated with either a fold or a cusp on the wave surface. Most of the caustic lines are symmetrical with respect to the principal symmetry plane and can be characterized by the caustic points at the centers of the caustic lines. These characteristic caustic points originated from inflection/parabolic points with zero in-plane/ex-plane curvature, respectively. By employing the Stroh formalism, the inflection/parabolic points on the slowness surface are studied in terms of the so-called zero-curvature transonic states. Since these transonic states are related to extraordinary degeneracies in the Stroh eigenvalue equation, the conditions for the degeneracies can be regarded as critical conditions for the characteristic caustic points. These conditions provide an overview of global structure of the phonon focusing patterns in tetragonal crystals. A set of caustic lines in vicinity of (001) plane is also investigated and exemplified. © 2010 Acoustical Society of America.

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I. INTRODUCTION

Acoustic wave propagation in anisotropic media is governed by the Christoffel equation, and the phase velocity is described by the slowness surface. Among three slowness sheets, two outer ones can be concave locally, and the so-called parabolic line, along which the Gaussian curvature is zero, divides the slowness sheet into concave, saddle, and convex regions. Because of the concavities, the wave surface, which describes the group velocity, will have some folds, and they result in various types of caustic lines in the phonon focusing pattern. The main characteristics of the phonon focusing pattern are therefore directly related to the geometry of the parabolic lines.

The phonon focusing pattern was first studied extensively in cubic crystals by examining the wave surface, and a complete set of critical existence conditions was formulated. Many numerical and experimental investigations were done in documenting the various patterns and their features. The understanding of the focusing patterns in tetragonal crystals, however, has been mainly based on numerical simulations and the focusing patterns were classified into 11 categories by comparing their features near the [001] direction to those in cubic crystals. By calculating the Gaussian curvature in the vicinity of two/fourfold symmetry axes, four criteria for negative Gaussian curvature, together with two parameters, were established for categorizing the focusing patterns in detail.

Recently, the present author developed an analytical scheme for finding locations of various caustic points in the symmetry planes in cubic crystals. The scheme is based on the so-called Stroh formalism for two-dimensional elastodynamics, where the zero-curvature points on the slowness surface are treated in terms of highly degenerated Stroh eigenvalues. The main idea for the scheme will be adopted in this investigation.

Generally, the phonon focusing patterns in tetragonal crystals consist of two types of caustic lines originated from fold/cusp on the wave surface. The existence of these caustic lines can be studied by concentrating on the central points of fold/cusp, which appear usually at principal symmetry planes. Let (±,±) denote the principal in-plane and ex-plane (transverse) curvature, respectively, with respect to a symmetry plane. Whenever a concave region appears on the slowness surface, the local geometry along the symmetry plane would evolve in following fashions:

(a) \((++)\rightarrow(+\rightarrow(-))\),
(b) \((++)\rightarrow(-\rightarrow(-))\), and
(c) \((++)\rightarrow(-\rightarrow(-))\).

The transition \((+\rightarrow(-))\) will take place at an inflection point with vanishing in-plane curvature, while the transition \((-\rightarrow(-))\) will occur at a parabolic point with vanishing transverse curvature. Case (c) refers to a rare case where an inflection point coalesces with a parabolic point, which is a monkey saddle point often found in the cubic crystals.

Figure 1 illustrates two segments of parabolic lines crossing a symmetry plane at these points and their corresponding caustic lines: The inflection point yields the central point of a parabola-formed caustic line associated with a fold on the wave surface [Fig. 1(a)], while the parabolic point produces the tip of two semicubical parabola-formed (swallowtail) caustic lines associated with a cusp on the wave surface [Fig. 1(b)]. We denote therefore these two caustic points at the symmetry plane as the cuspidal point and the swallowtail point, respectively.

In tetragonal crystals, there will be three sets of cuspidal (C/swallowtail (S) points along the principal symmetry planes \((100), \{110\}, \{001\}) Figure 2 illustrates the focusing pattern of InBi where the caustic points along the (010) plane \((C_{1}-S_{1}, C_{2}-S_{2})\) and the (110) plane \((C_{3}-C_{4}-S_{3})\) can be clearly identified. The central region of the pattern...